Midterm Exam Practise Paper

1 Solve the inequality $2 - 3x < |x - 3|$. [4]

1	*EITHER*:	State or imply non-modular inequality $(2 - 3x)^2 < (x - 3)^2$, or corresponding equation, and make a reasonable solution attempt at a 3-term quadratic	M1
		Obtain critical value $x = -\frac{1}{2}$	A1
		Obtain $x > -\frac{1}{2}$	A1
		Fully justify $x > -\frac{1}{2}$ as only answer	A1
	*OR*1:	State the relevant critical linear equation, i.e. $2 - 3x = 3 - x$	B1
		Obtain critical value $x = -\frac{1}{2}$	B1
		Obtain $x > -\frac{1}{2}$	B1
		Fully justify $x > -\frac{1}{2}$ as only answer	B1
	*OR*2:	Obtain the critical value $x = -\frac{1}{2}$ by inspection, or by solving a linear inequality	B2
		Obtain $x > -\frac{1}{2}$	B1
		Fully justify $x > -\frac{1}{2}$ as only answer	B1

2 The polynomial $2x^3 + ax^2 - 4$ is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $p(x)$, [2]

(iii) solve the inequality $p(x) > 0$, justifying your answer. [2]

2 (i)	Substitute 2 for x and equate to zero, or divide by $x - 2$ and equate remainder to zero	M1	
	Obtain answer $a = -3$	A1	**2**
(ii)	Attempt to find quadratic factor by division or inspection	M1	
	State quadratic factor $2x^2 + x + 2$	A1	**2**
	[The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c, or if two coefficients with the correct moduli are stated without working.]		
(iii)	State answer $x > 2$ (and nothing else)	B1*	
	Make a correct justification e.g. $2x^2 + x + 2$ (has no zeros and) is always positive	B1(dep*)	**2**
	[SR: The answer $x \geq 2$ gets B0, but in this case allow the second B mark if the remaining work is correct.]		

3 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

3	**(i)** State correct first two terms of the expansion of $(1+ax)^{\frac{2}{3}}$, i.e. $1+\frac{2}{3}ax$	B1	
	Form an expression for the coefficient of x in the expansion of $(1+2x)(1+ax)^{\frac{2}{3}}$ and equate it to zero	M1	
	Obtain $a = -3$	A1	**3**
	(ii) Obtain correct unsimplified terms in x^2 and x^3 in the expansion of $(1-3x)^{\frac{2}{3}}$ or $(1+ax)^{\frac{2}{3}}$	B1√ + B1√	
	Carry out multiplication by $1 + 2x$ obtaining two terms in x^3	M1	
	Obtain final answer $-\frac{10}{3}x^3$, or equivalent	A1	**4**
	[Symbolic binomial coefficients, e.g. $\binom{\frac{2}{3}}{1}$, are not acceptable for the B marks in **(i)** or **(ii)**]		

4 **(i)** Express $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in ascending powers of x, up to and including the term in x^2. [5]

4	**(i)** State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{1+2x}+\dfrac{C}{2+x}$	B1	
	Use any relevant method to determine a constant	M1	
	Obtain $A = 1$, $B = 2$ and $C = -4$	A1 + A1 + A1	**[5]**
	(ii) Use correct method to obtain the first two terms of the expansion of $(1-x)^{-1}$, $(1+2x)^{-1}$, $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$	M1	
	Obtain complete unsimplified expansions up to x^2 of each partial fraction	A1√ + A1√ + A1√	
	Combine expansions and obtain answer $1-2x+\frac{17}{2}x^2$	A1	**[5]**
	[Binomial coefficients such as $\binom{-1}{2}$ are not sufficient for the M1. The f.t. is on A, B, C.]		
	$2-x+8x^2 \quad 1-x \;^{-1} \quad 1+2x \;^{-1} \quad 2+x \;^{-1}$		
	[Apply this scheme to attempts to expand ( )( ) ( ) ( ) , giving M1A1A1A1 for the expansions, and A1 for the final answer.]		
	[Allow Maclaurin, giving M1A1√A1√ for f(0) = 1 and f ′(0) = –2, A1√ for f ″(0) = 17 and A1 for the final answer (f.t. is on A, B,C).]		

5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

5	**(i)**	Substitute $x = \frac{1}{2}$ and equate to zero, or divide, and obtain a correct equation, e.g. $\frac{1}{8}a + \frac{1}{4}b + \frac{5}{2} - 2 = 0$	B1	
		Substitute $x = 2$ and equate result to 12, or divide and equate constant remainder to 12	M1	
		Obtain a correct equation, e.g. $8a + 4b + 10 - 2 = 12$	A1	
		Solve for a or for b	M1	
		Obtain $a = 2$ and $b = -3$	A1	[5]
	(ii)	Attempt division by $2x - 1$ reaching a partial quotient $\frac{1}{2}ax^2 + kx$	M1	
		Obtain quadratic factor $x^2 - x + 2$	A1	[2]
		[The M1 is earned if inspection has an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B, or an unknown factor of $\frac{1}{2}ax^2 + Bx + C$ and an equation in B and/or C.]		

6 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

(i) Find the value of a and hence factorise $p(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\frac{8x - 13}{p(x)}$ in partial fractions. [5]

6	(i) Substitute $x = \frac{1}{2}$ and equate to zero			
	or divide by $(2x-1)$, reach $\frac{a}{2}x^2 + kx + ...$and equate remainder to zero			
	or by inspection reach $\frac{a}{2}x^2 + bx + c$ and an equation in b/c			
	or by inspection reach $Ax^2 + Bx + a$ and an equation in A/B		M1	
	Obtain $a = 2$		A1	
	Attempt to find quadratic factor by division or inspection or equivalent		M1	
	Obtain $(2x-1)(x^2+2)$		A1cwo	[4]
	(ii) State or imply form $\dfrac{A}{2x-1} + \dfrac{Bx+C}{x^2+2}$, following factors from part **(i)**		B1√	
	Use relevant method to find a constant		M1	
	Obtain $A = -4$, following factors from part **(i)**		A1√	
	Obtain $B = 2$		A1	
	Obtain $C = 5$		A1	

7 Expand $\dfrac{1}{(2+x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

: Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+x)^{-3}$ or $\left(1+\frac{1}{2}x\right)^{-3}$	M1
State correct first term $\frac{1}{8}$	B1
Obtain next two terms $-\frac{3}{16}x + \frac{3}{16}x^2$	A1 + A1
[The M mark is not earned by versions with unexpanded binomial coefficients such as $\binom{-3}{1}$.]	
[Accept exact decimal equivalents of fractions.]	
[SR: Answers given as $\frac{1}{8}\left(1-\frac{3}{2}x+\frac{3}{2}x^2\right)$ can earn M1B1A1.]	
[SR: Solutions involving $k\left(1+\frac{1}{2}x\right)^{-3}$, where k = 2, 8 or $\frac{1}{2}$, can earn M1 and A1√ for correctly simplifying both the terms in x and x^2.]	

8 Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}	M1
Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \dfrac{5}{1 - {}^1/_5}$	A1
Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$	M1
Obtain answer $x = 1.14$	A1

9 The variables x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $\ln y$ against x is a straight line.

(i) Find the gradient of this line. [2]

(ii) Given that the line intersects the axis of $\ln y$ at the point where $\ln y = 0.5$, find the value of A correct to 2 decimal places. [2]

(i)	State or imply $3\ln y = \ln A + 2x$ at any stage	B1	
	State gradient is $\dfrac{2}{3}$, or equivalent	B1	[2]
(ii)	Substitute $x = 0$, $\ln y = 0.5$ and solve for A	M1	
	Obtain $A = 4.48$	A1	[2]

10 Solve the equation $\ln(2 + e^{-x}) = 2$, giving your answer correct to 2 decimal places. [4]

State or imply $2 + e^{-x} = e^2$	B1	
Carry out method for finding $\pm x$ from $e^{\pm x} = k$, where $k > 0$, following sound ln or exp work	M1	
Obtain $x = -\ln(e^2 - 2)$, or equivalent expression for x	A1	
Obtain answer $x = -1.68$	A1	**4**
[The answer must be given to 2 decimal places]		
[SR: the M1 is available for attempts starting with $2 + e^{-x} = 10^2$]		

11 It is given that $\tan 3x = k \tan x$, where k is a constant and $\tan x \neq 0$.

(i) By first expanding $\tan(2x + x)$, show that

$$(3k - 1)\tan^2 x = k - 3.$$ [4]

(ii) Hence solve the equation $\tan 3x = k \tan x$ when $k = 4$, giving all solutions in the interval $0° < x < 180°$. [3]

(iii) Show that the equation $\tan 3x = k \tan x$ has no root in the interval $0° < x < 180°$ when $k = 2$. [1]

11	**(i)**	Use $\tan(A + B)$ and $\tan 2A$ formulae to obtain an equation in $\tan x$	M1	
		Obtain a correct equation in $\tan x$ in any form	A1	
		Obtain an expression of the form $a\tan^2 x = b$	M1	
		Obtain the given answer	A1	[4]
	(ii)	Substitute $k = 4$ in the given expression and solve for x	M1	
		Obtain answer, e.g. $x = 16.8°$	A1	
		Obtain second answer, e.g. $x = 163.2°$, and no others in the given interval	A1	[3]
		[Ignore answers outside the given interval. Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
	(iii)	Substitute $k = 2$, show $\tan^2 x < 0$ and justify given statement correctly	B1	[1]

12 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \dfrac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \tfrac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

12 **(i)**	Use $\sin(A+B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
	Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
	Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
	Obtain the given identity	A1	**[4]**
	[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]		
(ii)	Substitute for x and obtain the given answer	B1	**[1]**
(iii)	Carry out a correct method to find a value of x	M1	
	Obtain answers 0.322, 0.799, –1.12	A1 + A1 + A1	**[4]**
	[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]		

13 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
Divide by $\cos A$ to find value of $\tan A$	**M1**	
Obtain $\tan A = 3$	**A1**	
Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
Solve three-term quadratic equation and find $\tan B$	**M1**	
Obtain $\tan B = \dfrac{3}{2}$ only	**A1**	
Substitute **numerical values** in $\dfrac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
Obtain $\dfrac{3}{11}$	**A1**	[8]

14 Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x. \qquad [3]$$

EITHER	Make relevant use of the correct sin 2*A* formula	M1
	Make relevant use of the correct cos 2*A* formula	M1
	Derive the given result correctly	A1
OR	Make relevant use of the tan 2*A* formula	M1
	Make relevant use of $1 + \tan^2 A = \sec^2 A$ or $\cos^2 A + \sin^2 A = 1$	M1
	Derive the given result correctly	A1

15 Solve the equation

$$\cos\theta + 3\cos 2\theta = 2,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

Use correct cos 2*A* formula, or equivalent pair of correct formulas, to obtain an equation in cos θ	M1
Obtain 3-term quadratic $6\cos^2\theta + \cos\theta - 5 = 0$, or equivalent	A1
Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$	M1
Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians	A1
Obtain answer 180° or π (or 3.14) radians and no others in range	A1

16 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

(i)	State answer $R = 25$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 73.74°$	A1	3
(ii)	Carry out evaluation of $\cos^{-1}(15/25)$ $(= 53.1301...°)$	M1	
	Obtain answer 126.9°	A1	
	Carry out correct method for second answer	M1	
	Obtain answer 20.6° and no others in the range	A1√	4
	[Ignore answers outside the given range.]		